1. **Logical Equivalences**

Use truth tables to determine whether the following claimed logical equivalences are true:

1. **𝑃 ∨ 𝑃 ⇔ 𝑃**

|  |  |
| --- | --- |
| **p** | **p∨p** |
| F | F |
| T | T |

Both truth tables gives the same output. Therefore, the logical equivalence is ***TRUE***

1. **𝑃 → (𝑃 ∨ 𝑄) ⇔ (¬ 𝑃) → (𝑃 → 𝑄)**

|  |  |  |
| --- | --- | --- |
| **p** | **q** | **p→(p∨q)** |
| F | F | T |
| F | T | T |
| T | F | T |
| T | T | T |

|  |  |  |
| --- | --- | --- |
| **p** | **q** | **(¬p)→(p→q)** |
| F | F | T |
| F | T | T |
| T | F | T |
| T | T | T |

Both truth tables gives the same output. Therefore, the logical equivalence is ***TRUE***

1. **(𝑃 ∧ (𝑃 → 𝑄)) ⇔ 𝑄**

|  |  |  |
| --- | --- | --- |
| **p** | **q** | **(p∧(p→q))⇔q** |
| F | F | T |
| F | T | F |
| T | F | T |
| T | T | T |

Both truth tables does not give the same output. Therefore, the logical equivalence is ***FALSE***

1. **𝑃 ↔ 𝑄 ⇔ ¬(𝑄 ⊕ 𝑃)**

|  |  |  |
| --- | --- | --- |
| **p** | **q** | **p↔q** |
| F | F | T |
| F | T | F |
| T | F | F |
| T | T | T |

|  |  |  |
| --- | --- | --- |
| **q** | **p** | **¬(q⊕p)** |
| F | F | T |
| F | T | F |
| T | F | F |
| T | T | T |

Both truth tables gives the same output. Therefore, the logical equivalence is ***TRUE***

1. **𝑃 ∧ (𝑄 ∨ 𝑅) ⇔ (𝑃 ∧ 𝑄) ∨ 𝑅**

|  |  |  |  |
| --- | --- | --- | --- |
| **p** | **q** | **r** | **p∧(q∨r)** |
| F | F | F | F |
| F | F | T | F |
| F | T | F | F |
| F | T | T | F |
| T | F | F | F |
| T | F | T | T |
| T | T | F | T |
| T | T | T | T |

|  |  |  |  |
| --- | --- | --- | --- |
| **p** | **q** | **r** | **(p∧q)∨r** |
| F | F | F | F |
| F | F | T | T |
| F | T | F | F |
| F | T | T | T |
| T | F | F | T |
| T | F | T | T |
| T | T | F | T |
| T | T | T | T |

Both truth tables does not give the same output. Therefore, the logical equivalence is ***FALSE***

1. **𝑃 ∧ (𝑄 ∨ 𝑅) ⇔ (𝑃 ∧ 𝑄) ∨ (𝑃 ∧ 𝑅)**

|  |  |  |  |
| --- | --- | --- | --- |
| **p** | **q** | **r** | **p∧(q∨r)** |
| F | F | F | F |
| F | F | T | F |
| F | T | F | F |
| F | T | T | F |
| T | F | F | F |
| T | F | T | T |
| T | T | F | T |
| T | T | T | T |

|  |  |  |  |
| --- | --- | --- | --- |
| **p** | **q** | **r** | **(p∧q)∨(p∧r)** |
| F | F | F | F |
| F | F | T | F |
| F | T | F | F |
| F | T | T | F |
| T | F | F | F |
| T | F | T | T |
| T | T | F | T |
| T | T | T | T |

Both truth tables gives the same output. Therefore, the logical equivalence is ***TRUE***

1. **𝑃 ∨ (¬𝑃) ⇔ 𝑇𝑟𝑢𝑒**

|  |  |  |
| --- | --- | --- |
| **𝑃** | **¬𝑃** | **𝑃 ∨ (¬𝑃)** |
| F | T | T |
| T | F | T |

Both truth tables gives the same output. Therefore, the logical equivalence is ***TRUE***

1. **Logical Implication**

Use truth tables to prove the following:

1. **¬P ⇒ P → Q**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **𝑃** | **Q** | **¬𝑃** | **P → Q** | **(¬P ⇒ (P → Q))** |
| T | F | T | T | T |
| T | T | T | T | T |
| F | T | F | T | T |
| F | T | F | F | T |

1. **((P → Q) ∧(Q → R)) ⇒ P → R**

|  |  |  |  |
| --- | --- | --- | --- |
| **P** | **Q** | **R** | **((P→Q) ∧ (Q→R))** |
| F | F | F | T |
| F | F | T | T |
| F | T | F | F |
| F | T | T | T |
| T | F | F | F |
| T | F | T | F |
| T | T | F | F |
| T | T | T | T |

|  |  |  |  |
| --- | --- | --- | --- |
| **P** | **Q** | **R** | **(((P → Q) ∧ (Q → R)) ⇒ (P → R))** |
| F | F | F | T |
| F | F | T | T |
| F | T | F | T |
| F | T | T | T |
| T | F | F | T |
| T | F | T | T |
| T | T | F | T |
| T | T | T | T |

1. **Transformational Proof using Logical Equivalences**
2. 𝑃 → (¬𝑃) ⇔ (¬𝑃)

𝑃 → (¬𝑃)

⇔ **(¬𝑃) V (¬𝑃)** implication law

⇔ ¬𝑃 **Idempotent law**

1. (𝑃 → (𝑃 ∧ 𝑄)) ⇔ (𝑃 → 𝑄)

𝑃 → (𝑃 ∧ 𝑄)

⇔ **(¬𝑃) V (𝑃 ∧ 𝑄)**  implication law

⇔ ((¬𝑃) ∨ 𝑃) ∧ ((¬𝑃) ∨ 𝑄) **Distributive law**

⇔ 𝑇𝑟𝑢𝑒 ∧ ((¬𝑃) ∨ 𝑄) **Excluded Middle**

⇔ (¬𝑃) ∨ 𝑄 **Identity law**

⇔ 𝑃 → 𝑄 **Implication law**

1. ((¬𝑃) → (𝑃 → 𝑄)) ⇔ 𝑇𝑟𝑢𝑒

(¬𝑃) → (𝑃 → 𝑄)

⇔ (¬(¬𝑃)) ∨ (𝑃 → 𝑄) implication law

⇔ **P V (P→ Q)** double negation law

⇔ 𝑃 ∨ ((¬𝑃) ∨ 𝑄) **Implication law**

⇔ **(𝑃 ∨ (¬𝑃)) ∨ 𝑄** associative law

⇔ **True ∨ 𝑄** excluded middle law

⇔ **True**  **Domination law**

1. (𝑃 ∨ 𝑄) ∧ ((¬𝑃) ∨ 𝑄) ⇔ 𝑄

(𝑃 ∨ 𝑄) ∧ ((¬𝑃) ∨ 𝑄)

⇔ (𝑄 ∨ 𝑃) ∧ ((¬𝑃) ∨ 𝑄) **Commutative law**

⇔ (𝑄 ∨ 𝑃) ∧ (𝑄 ∨ (¬𝑃)) **Commutative law**

⇔ 𝑄 ∨ (𝑃 ∧ (¬𝑃)) **Distributive law**

⇔ **Q V FALSE** **Contradiction law**

⇔ **Q**  **Identity law**

1. **State all the models for the following sets of propositional formulae.**
2. { (( ¬P) ∨ Q), ( P ∨ Q), ( P → Q )}

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **p** | **q** | **¬p** | **(¬p)∨q** | **p∨q** | **p→q** |
| F | F | T | T | F | T |
| F | T | T | T | T | T |
| T | F | F | F | T | F |
| T | T | F | T | T | T |

1. { ( P ∧ ( ¬Q)), ( P ∨ Q), ( P**→**Q )}

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **p** | **q** | **¬q** | **p∧(¬q)** | **p∨q** | **p→q** |
| F | F | T | F | F | T |
| F | T | F | F | T | T |
| T | F | T | T | T | F |
| T | T | F | F | T | T |

1. { ( P ∧ Q), ( Q ∨ R), ( P → Q), ( R → Q )}

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p∧q** | **q∨r** | **p→q** | **r→q** |
| F | F | F | F | F | T | T |
| F | F | T | F | T | T | F |
| F | T | F | F | F | F | T |
| F | T | T | F | T | F | T |
| T | F | F | F | T | T | T |
| T | F | T | F | T | T | T |
| T | T | F | F | T | T | F |
| T | T | T | T | T | T | T |

1. { ( P → Q), ( Q → R), ( ¬ ( P → R) )}

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p→q** | **q→r** | **p→r** | **(¬ (P → R))** |
| F | F | F | T | T | T | F |
| F | F | T | T | T | T | F |
| F | T | F | T | F | T | F |
| F | T | T | T | T | T | F |
| T | F | F | F | T | F | T |
| T | F | T | F | T | T | F |
| T | T | F | T | F | F | T |
| T | T | T | T | T | T | F |

1. **Proving validity of an argument using truth tables**

{(𝑃 → 𝑄), (𝑄 → 𝑅)} ⊢ (R → Q)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **# of Rows** | **p** | **q** | **r** | **p→q** | **q→r** | **\*C (r→q)** |
| 1 | F | F | F | T | T | T |
| 2 | F | F | T | T | T | F |
| 3 | F | T | F | T | F | T |
| 4 | F | T | T | T | T | T |
| 5 | T | F | F | F | T | T |
| 6 | T | F | T | F | T | F |
| 7 | T | T | F | T | F | T |
| 8 | T | T | T | T | T | T |

The argument is INVALID since the conclusion of the model are not all true

Conclusion False in Row 2

{(P ∧ Q), (Q → (¬R)), (S → R)} ⊢ ((¬S) ∧ P)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **# of Rows** | **p** | **q** | **r** | **s** | **p∧q** | **¬r** | **q → (¬r)** | **s → r** | **¬s** | **\*C (¬s)∧p** |
| 1 | F | F | F | F | F | T | T | T | T | F |
| 2 | F | F | F | T | F | T | T | F | F | F |
| 3 | F | F | T | F | F | F | T | T | T | F |
| 4 | F | F | T | T | F | F | T | T | F | F |
| 5 | F | T | F | F | F | T | T | T | T | F |
| 6 | F | T | F | T | F | T | T | F | F | F |
| 7 | F | T | T | F | F | F | F | T | T | F |
| 8 | F | T | T | T | F | F | F | T | F | F |
| 9 | T | F | F | F | F | T | T | T | T | T |
| 10 | T | F | F | T | F | T | T | F | F | F |
| 11 | T | F | T | F | F | F | T | T | T | T |
| 12 | T | F | T | T | F | F | T | T | F | F |
| 13 | T | T | F | F | T | T | T | T | T | T |
| 14 | T | T | F | T | T | T | T | F | F | F |
| 15 | T | T | T | F | T | F | F | T | T | T |
| 16 | T | T | T | T | T | F | F | T | F | F |

The argument is VALID since the conclusion of the model is all True